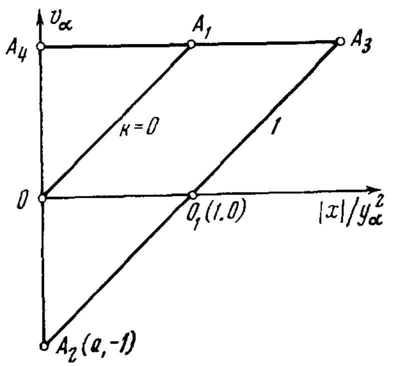

Fig. 3

$$u^\circ = -\mu\delta, \qquad v^\circ = +1 = v_\alpha$$

and contact takes place at the instant

$$T[u^\circ, v^\circ] = (1/2)\,[-(y_\alpha - \mu) - \sqrt{(y_\alpha - \mu)^2 - 4x}]$$

in the region $W^\circ\,[y_\alpha - \mu \leqslant 2\sqrt{x}]$. In case $k = 1$ [4] the optimal control is $v^\circ = v_\alpha = +1$ on the whole set W°. At positions of the one-dimensional analogs, which in the sense of the equalities

$$|x_{(1)}| = |x|, \quad y_\beta = 0, \quad y_\alpha = y_{(1)}, \quad \mu = \mu_{(1)}$$

duplicate the positions in the three-dimensional problem, the optimal time is strictly less than the optimal time in the three-dimensional problem. This fact is explained by the absence of a lateral maneuver in the one-dimensional problem.

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EXACT FORMULAS IN THE CONTROL PROBLEM OF CERTAIN SYSTEMS WITH AFTEREFFECT

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We examine the problem of the optimal control of linear systems with aftereffect and with quadratic performance criterion. We distinguish the class of such systems for which the corfficients of the optimal control and of the functional to be minimized are computed in explicit form.

1. Let there be given the controlled system

$$x^{\cdot}(t) = A(t)\,x(t) + B(t)\,x(t-h) + D(t)\,u(t), \qquad 0 \leqslant t \leqslant T \quad (1.1)$$

Here the vector $x(t)$ belongs to an Euclidean space R_n of dimension n, the control $u(t) \in R_m$, the constant $h \geqslant 0$, and A, B, D are given matrices with piecewise-continuous elements. The solution of system (1.1) is determined by the initial conditions

$$x(\tau) = \varphi(\tau), \qquad -h \leqslant \tau \leqslant 0 \tag{1.2}$$

where the specified measurable bounded function $\varphi(\tau) \in R_n$.

Problem 1. Find the control $u(t)$ minimizing the functional

$$x'(T) H x(T) + \int_0^T [x'(s) N_1(s) x(s) + u'(s) N(s) u(s)]\, ds \tag{1.3}$$

on the trajectories $x(t)$ of system (1.1), (1.2). Here the prime is the transpose sign, the elements of matrices $N_1(s)$, $N(s)$ are piecewise-continuous functions, the matrices H and $N_1(s)$ are nonnegative definite, while matrix $N(s)$ is positive definite for all $s \in [0, T]$.

A number of papers have been devoted to the investigation of Problem 1. The results of [1, 2], in which a general approach to the solution of Problem 1 has been outlined, show, in particular, that in the general case it is difficult to determine explicitly the coefficients of the optimal control. In this connection a number of subsequent investigations were devoted to the approximation of Problem 1. The possibility was investigated of approximating Problem 1 by a controlled system of ordinary differential equations [3–5]. Method has been proposed for the construction of the optimal control with the aid of a "deformation" of system (1.1) [1]. Another possible method for the approximate study of Problem 1 consists in the construction of approximations of the optimal control. The form of the successive Bellman approximations was established in [6] for $H = 0$. For general linear stochastic systems with aftereffect and with an arbitrary matrix H, in [7] there were established the formulas for the successive approximations to the optimal control and to the functional, the convergence of the successive approximations, the limit partial differential equations for the coefficients of the optimal control, as well as the existence conditions, in terms of the control system parameters, for the solution in these limit equations.

In connection with the results in [1–7] there arises the problem of determining the class of controlled systems (1.1) – (1.3) for which the coefficients of the optimal control and of the functional (1.3) corresponding to this control are computed in explicit form. Below, these coefficients have been computed for system (1.1) – (1.3) with $N_1(t) \equiv 0$, and next, for stochastic systems. With respect to the constraint $N_1(t) \equiv 0$ we note that when $N_1(t) \neq 0$ the equations for the optimal coefficients are, in general, not integrable in explicit analytic form even when $h = 0$.

2. Everywhere in what follows it is assumed that $N_1(t) \equiv 0$, that the coefficients in (1.1), (1.3) satisfy the requirements in Sect. 1, and that the elements of matrix $B(t)$ are piecewise-continuously differentiable. Then (see [7]) the following results are valid. The optimal control $u_0(t)$ in Problem 1 is

$$u_0(t) = -N^{-1}(t) D'(t) \Big[P(t) x(t) + \int_{-h}^{0} Q(t, \tau) x(t+\tau)\, d\tau \Big] \tag{2.1}$$

and the value, corresponding to control (2.1), of functional (1.3) in which the lower limit of integration is t equals

$$I(t) = \varphi'(t) P(t) \varphi(t) + \varphi'(t) \int_{-h}^{0} Q(t, \tau) \varphi(t+\tau)\, d\tau + \tag{2.2}$$

$$\int_{-h}^{0}\varphi'(t+\tau)\,Q'(t,\tau)\,d\tau\varphi(t)+\int_{-h}^{0}\int_{-h}^{0}\varphi'(t+\tau)\,R(t,\tau,\rho)\,\varphi(\rho+t)\,d\tau d\rho$$

on the trajectories of system (1.1) under the initial conditions

$$x(\sigma)=\varphi(\sigma),\quad t-h\leqslant\sigma\leqslant t$$

In particular, the value of functional (1.3) with control (2.1) is

$$I_0=I(0)$$

The matrices P, Q, R satisfy almost everywhere the equations

$$P^{\cdot}(t)+A'(t)\,P(t)+P(t)\,A(t)+Q(t,0)+Q'(t,0)=P(t)\,D_1(t)P(t)\quad(2.3)$$

$$A'(t)\,Q(t,\tau)+R(t,0,\tau)+\frac{\partial Q(t,\tau)}{\partial t}-\frac{\partial Q(t,\tau)}{\partial\tau}=P(t)\,D_1(t)\,Q(t,\tau)$$

$$\frac{\partial R(t,\tau,\rho)}{\partial t}-\frac{\partial R(t,\tau,\rho)}{\partial\tau}-\frac{\partial R(t,\tau,\rho)}{\partial\rho}=Q'(t,\tau)\,D_1(t)\,Q(t,\rho)$$

$$0\leqslant t\leqslant T,\quad -h\leqslant\tau,\ \rho\leqslant 0,\ D_1(t)=D(t)\,N^{-1}(t)\,D'(t)$$

Here $N^{-1}(t)$ is the matrix inverse to $N(t)$. The boundary conditions for system (2.3) are

$$P(T)=H,\quad Q(T,\tau)=R(T,\tau,\rho)\equiv 0,\quad -h<\tau,\quad \rho\leqslant 0$$
$$B'(t)\,P(t)-Q'(t,-h)=0,\qquad 0\leqslant t\leqslant T\qquad(2.4)$$
$$2B'(t)\,Q(t,\tau)-R(t,-h,\tau)-R'(t,\tau,-h)=0$$

Hence, the synthesis problem of the optimal control for system (1.1) is solved if we construct the solution of the boundary value problem (2.3),(2.4). Note that for $h>0$ the first of the equations in system (2.3) is a Riccati matrix differential equation. Let us state the answer. We denote the matrix $z(t)$ as follows:

$$z(t)=\exp\int_{0}^{t}A(s)\,ds$$

We define a matrix $B_1(t)$ as the solution of the Cauchy problem

$$B_1^{\cdot}(t)=-B_1(t+h)\,z(t+h)^{-1}B(t+h)\,z(t),\quad 0\leqslant t\leqslant T$$
$$B_1(T)=I,\qquad B_1(t)\equiv 0,\qquad t>T\qquad(2.5)$$

Here I is the unit matrix, while the matrix $P_1(t)$ is given by the relations

$$P_1^{\cdot}(t)=P_1(t)\,B_1(t)\,z(t)^{-1}D_1(t)\,z'(t)^{-1}B_1'(t)\,P_1(t)$$
$$P_1(T)=z'(T)Hz(T)\qquad(2.6)$$

Then the coefficients P, Q, R of the optimal control and the optimal value of functional (1.3) are

$$P(t)=z'(t)^{-1}B_1'(t)P_1(t)B_1(t)\,z(t)^{-1}$$
$$Q(t,\tau)=-z'(t)^{-1}B_1'(t)\;P_1(t)\,B_1^{\cdot}(t+\tau)\,z(t+\tau)^{-1}\qquad(2.7)$$
$$R(t,\tau,\rho)=z'(t+\tau)^{-1}\dot{B}_1'(t+\tau)\,P_1(t)\,\dot{B}_1(t+\rho)\,z(t+\rho)^{-1}$$

At the unique point $t=T-h$ of discontinuity of the first kind of the derivative

$B_1^{\cdot}(t)$; this derivative is defined by continuity from the left. The validity of these relations for P, Q, R can be verified by a direct substitution of them into (2.3), (2.4). Thus, the functions P, Q, R are determined in terms of matrices $B_1(t)$ and $P_1(t)$.

To compute the matrix $B_1(t)$ we should integrate system (2.5) from the point $t = T$ to $t = 0$. Using the method of mathematical induction it is not difficult to obtain that the matrix

$$B_1(t) = I + \int_t^{T-h} f(s)\,ds + \int_t^{T-2h} f(s)\,ds \int_{s+h}^{T-h} f(s_1)\,ds_1 + \ldots + \int_t^{T-jh} f(s)\,ds \int_{s+h}^{T-jh+h} f(s_1)\,ds_1 \ldots \int_{s_{j-2}+h}^{T-h} f(s_{j-1})\,ds_{j-1}$$

for $T-(j+1)h \leqslant t \leqslant T-jh$ (the integer $j \geqslant 1$). Here, by virtue of (2.5), the matrix $f(t)$ is

$$f(t) = z(t+h)^{-1} B(t+h)\, z(t)$$

It is clear that $B_1(t) \equiv I$ for $T-h \leqslant t \leqslant T$. By the same token, $B_1(t)$ has been computed.

We note further that the matrix $I + \alpha$ is positive definite for any nonnegative-definite matrix α. From this and from (2.6) we conclude that

$$P_1(t) = \left[I + z'(T)\,Hz(T)\int_t^T B_1(s)\,z(s)^{-1}D_1(s)\,z'(s)^{-1}B_1'(s)\,ds\right]^{-1} z'(T)\,Hz(T)$$

3. Let us cite a method for obtaining formula (2.7). We note that the indicated method does not depend upon the dimension of system (1.1). Therefore, we restrict ourselves to presenting it only for the simplest scalar system (1.1) of the form

$$x^{\cdot}(t) = x(t-h) + u(t), \quad 0 \leqslant t \leqslant T, \quad x(\tau) = \varphi(\tau), \quad -h \leqslant \tau \leqslant 0 \tag{3.1}$$

with a functional (1.3) equal to

$$x^2(T) + \int_0^T u^2(s)\,ds \tag{3.2}$$

At first we state auxiliary assertions needed in the examination of Problem 1 for system (1.1) – (1.3). Let a vector $y(t) \in R_n$ satisfy the equation

$$y^{\cdot}(t) = A(t)\,y(t) + f(t) + D(t)\,v(t), \quad y(0) = y_0, \quad 0 \leqslant t \leqslant T \tag{3.3}$$

Here the matrices A and D are the same as in (1.1), while $f(t)$ is a given measurable bounded function. For system (3.3) we examine Problem 1 with the functional (1.3) to be minimized, in which $y(t)$ takes the place of $x(t)$ and $v(t)$, of $u(t)$. A standard application of the dynamic programing principle in terms of the existence of Liapunov functions in system (3.3) shows that the following lemma is valid (cf [8]).

Lemma. The optimal control $v_0(t)$ in Problem 1 for system (3.3), (1.3) is

$$v_0(t) = -N^{-1}(t)\,D'(t)\,[r(t)\,y(t) + g'(t)] \tag{3.4}$$

The value, corresponding to the control $v_0(t)$, of functional (1.3) is

$$y_0'\,r(0)\,y_0 + g(0)\,y_0 + y_0'g'(0) + q(0) \tag{3.5}$$

In the last two formulas the matrix $r(t)$, the vector $g(t) \in R_n$, and the scalar function $g(t)$ are determined from the equations

$$r^{\cdot}(t) + A'(t)\,r(t) + r(t)\,A(t) - r(t)\,D_1(t)\,r(t) + N_1(t) = 0$$
$$r(T) = H \tag{3.6}$$

$$g^{\cdot}(t) + f'(t)\,r(t) + g(t)\,A(t) - g(t)\,D_1(t)\,r(t) = 0, \quad r(T) = 0$$

$$q^{\cdot}(t) + g(t)\,f(t) + f'(t)\,g'(t) - g(t)\,D_1(t)\,g(t) = 0, \quad q(T) = 0$$

Let us now return to the determination of the functions P, Q, R in Problem 1 for system (3.1), (3.2). On the basis of the dynamic programing principle

$$\min_{u(t),\, 0 \leqslant t \leqslant T} \left[x^2(T) + \int_0^T u^2(s)\,ds\right] = \tag{3.7}$$

$$\min_{u(t),\, 0 \leqslant t \leqslant T-h} \left[\int_0^{T-h} u^2(s)\,ds + \min_{u(t),\, T-h \leqslant t \leqslant T} \left((x^2(T) + \int_{T-h}^{T} u^2(t)\,dt\right)\right]$$

Let us assume that the optimal control $u(t)$ and the corresponding trajectory $x(t)$ of Eq. (3.1) has been found on the interval $[0, T-h]$. Then, to determine the optimal control $u(t)$ on the interval $[T-h, T]$ it is sufficient, in view of (3.1), (3.2), (3.7), to solve Problem 1 for the ordinary equation without aftereffect

$$y^{\cdot}(t) = f(t) + u(t), \quad y(T-h) = x(T-h), \quad T-h \leqslant t \leqslant T$$

with the functional

$$y^2(T) + \int_{T-h}^{T} u^2(t)\,dt$$

where the known function $f(t)$ equals $x(t-h)$ for $T-h \leqslant t \leqslant T$. On the basis of the lemma, relations (3.4) – (3.6) supply the solution to the latter problem. On the other hand, in correspondence with (2.1), (2.2), the functions P, Q, R also yield a solution of this problem. We then equate the values of the optimal control obtained by these two methods. Carrying out simple but cumbersome computations, we obtain that for $T - h \leqslant t \leqslant T$, $-h \leqslant \tau \leqslant 0$ the function $P(t) = r(t)$, while $Q(t, r) = 0$ if $t + \tau > T - h$ and

$$Q(t, \tau) = r(t + \tau + h) \exp \int_{t+\tau+h}^{t} r(s)\,ds, \quad t + \tau \leqslant T - h$$

Let us transform the expression for $Q(t, \tau)$ at $t + \tau \leqslant T - h$. On the basis of (3.1), (3.6) the equation for $r(t)$ can be written as

$$r^{\cdot}(t) = r^2(t), \quad r(T) = 1$$

Hence, the function $r(t) > 0$ for $T - h \leqslant t \leqslant T$; by integrating, we have

$$Q(t, \tau) = r(t + \tau + h) \exp \int_{t+\tau+h}^{t} r^{\cdot}(s)\,r^{-1}(s)\,ds = r(t), \quad t + \tau \leqslant T - h$$

In analogous manner, by equating to each other the values of the functional to be minimized, obtained by the two methods, we have

$$R(t, \tau, \rho) = r(t), \; (T - h \leqslant t \leqslant T)$$

in the region $t+\tau \leqslant T-h$, $t+\rho \leqslant T-h$, $-h \leqslant \tau, \rho \leqslant 0$, while $R(t, \tau, \rho) \equiv 0$ outside this region. Thus, the coefficients P, Q, R have been determined on the interval $[-h+T, T]$. Consequently, to solve Problem 1 for system (3.1), (3.2) it suffices to find the control which on the basis of (3.7) and of the values of $P(T-h)$, $Q(T-h, \tau)$, $R(T-h, \tau, \rho)$, supplies a minimum to the functional

$$\int_0^{T-h} u^2(t)\,dt + P(T-h)\Big(x^2(T-h) + 2\int_{T-2h}^{T-h} x(t)\,dt + \int_{T-2h}^{T-h}\int_{T-2h}^{T-h} x(s)x(t)\,ds\,dt\Big) \tag{3.8}$$

By similar reasonings, applied on $[T-h, T]$, to determine the optimal control for $T-2h \leqslant t \leqslant T-h$ it suffices to find the control $u(t)$ which, in view of (3.8), supplies a minimum to the functional

$$\int_{T-2h}^{T-h} u^2(t)\,dt + P(T-h)\Big(x(T-h) + \int_{T-2h}^{T-h} x(t)\,dt\Big)^2 \tag{3.9}$$

on the trajectories of system (3.1) with an arbitrary initial function $x(\tau)$, $T-3h \leqslant \tau \leqslant T-2h$.

The comparison method used directly on $[T-h, T]$ is no longer applicable since the last term in (3.9) is not the square of the coordinate at the last instant. This difficulty can be avoided by setting

$$y(t) = x(t)(1+T-h-t) + \int_{T-2h}^{t} x(s)\,ds \tag{3.10}$$

From (3.1), (3.10) it follows that $y(t)$ is determined by the equation

$$y^{\cdot}(t) = (x(t-h) + u(t))\,(1+T-t-h),\quad T-2h \leqslant t \leqslant T-h \tag{3.11}$$

and by virtue of (3.10), functional (3.9) equals

$$\int_{T-2h}^{T-h} u^2(t)\,dt + P(T-h)\,y^2(T-h) \tag{3.12}$$

Using the lemma, let us find the solution of Problem 1 for the system (3.11), (3.12) (recall that $x(t-h)$ in (3.11) is reckoned to be a known function of time). It is also obvious that the minimum value of functional (3.12) on the trajectories of (3.11) coincides with the minimum value of functional (3.9) on the trajectories of system (3.1). We now replace the function $y(t)$ in the value of the optimal control found by means of the lemma, by the right-hand side of (3.10) and we equate the result (2.1). We then get that for $T-2h \leqslant t \leqslant T-h$, $-h \leqslant \tau \leqslant 0$

$$P(t) = (1+T-h-t)^2 r(t) \tag{3.13}$$

$$Q(t,\tau) = r(t)\,(1+T-h-t),\quad \text{if}\quad t+\tau > T-2h$$

$$Q(t,\tau) = (1+T-2h-t-\tau)\,r(t+\tau+h)\,(1+T-h-t)\times$$

$$\exp\int_{t+\tau+h}^{t} r(s)(1+T-h-s)^2\,ds,\quad t+\tau \leqslant T-2h$$

We transform the expression for $Q(t, \tau)$ for $t + \tau \leqslant T - 2h$, using Eq. (3.6) defining $r(t)$. By virtue of (3.11) this equation can be written in the form

$$r^{\boldsymbol{\cdot}}(t) = r^2(t)(1 + T - h - t)^2, \quad r(T - h) = P(T - h)$$

From this and from (3.13) we conclude that for $t + \tau \leqslant T - 2h$

$$Q(t, \tau) = (1 + T - 2h - t - \tau)(1 + T - h - t)\, r(t + \tau + h) \times$$
$$\exp \int_{t+\tau+h}^{t} r^{\boldsymbol{\cdot}}(s)\, r(s)^{-1}\, ds = (1 + T - 2h - t - \tau)(1 + T - h - t)\, r(t)$$

We then equate the values of the functional to be minimized, obtained by the two methods (see formulas (3.5), (2.2)). Replacing in (3.5) the function $y(t)$ by the right-hand side of (3.10), we get that $R(t, \tau, \rho)$ can be written in the form (2.7) for $T - 2h \leqslant t \leqslant T - h, \ -h \leqslant \tau, \ \rho \leqslant 0$. By the same token the functions P, Q, R have been determined on the interval $[T - 2h, T - h]$. From the established form of the functions P, Q, R it follows that they admit of the representation (2.5) – (2.7) for $T - 2h \leqslant t \leqslant T$. In analogous manner, using the dynamic programing principle, the lemma, and a suitable change of variables of form (3.10), we can convince ourselves of the validity of representation (2.5) – (2.7) on the interval $T - 3h \leqslant t \leqslant T$. However, the validity of relations (2.7) on the whole interval $0 \leqslant t \leqslant T$ can be verified by a direct substitution of (2.7) into (2.3), (2.4).

4. Note 1. Using the results of [7] on the form of the partial differential equations for the coefficients of the optimal control in the case of general linear stochastic systems, as well as the methods for solving them, proposed in Sect. 2, we can establish formulas for these coefficients for certain stochastic systems with several discrete time lags.

As an example we cite the solution of the synthesis problem of the optimal control for the stochastic system

$$x^{\boldsymbol{\cdot}}(t) = A(t)\, x(t) + B(t)\, x(t - h) + D(t)\, u(t) + \sigma(t)\, \xi(t) \tag{4.1}$$

which is the system (1.1) subjected to the action of a Gaussian white noise $\xi(t)$ of intensity $\sigma(t)\sigma'(t)$, where $\sigma(t)$ is a given matrix with piecewise-continuous elements. We are required to minimize the functional

$$M\left(x'(T)\, Hx(T) + \int_0^T u'(s)\, N(s)\, u(s)\, ds\right) \tag{4.2}$$

(where M denotes the mean) on the trajectories of (4.1), (1.2) by choosing the control $u(t)$. Similarly to Sect. 2, using [7], we obtain that the optimal control in the problem posed is determined by formula (2.1), while the corresponding value of functional (4.2) is

$$I_0 + \int_0^T \operatorname{Tr} P(t)\, \sigma(t)\, \sigma'(t)\, dt$$

Here I_0 is given by equality (2.2), matrices P, Q, R are determined by relations (2.5)–(2.7), $\operatorname{Tr} A$ is the trace of matrix A.

Note 2. Let us further indicate the types of partial differential equations of type (2.3) whose solution can be found in explicit form and which are encountered in the investigation of certain controlled systems with aftereffect.

1) Matrices P, Q, R, specified by (2.5) – (2.7), are the solution of boundary

value problem (2.3), (2 4) also in the case when the last of requirements (2.4) has the form

$$(0 \leqslant t \leqslant T, \quad -h < \tau \leqslant 0)$$

$$B'(t)\, Q(t, \tau) + Q'(t, \tau)\, B(t) - R(t, -h, \tau) - R(t, \tau - h) = 0$$

2) Let there be given the scalar piecewise-continuous functions $A(t)$, $D(t), N(t)$, the piecewise-continuously differentiable function $B(t)$, and the constant $H \geqslant 0$. We are required to determine the scalar functions $P(t), Q(t, \tau), R(t, \tau, \rho), 0 \leqslant t \leqslant T, \; -h \leqslant \tau, \rho \leqslant 0$, satisfying the equations

$$P^{\cdot}(t) + 2A(t)\, P(t) + 2Q(t, 0) = P^2(t)\, D_1(t)$$

$$2A(t)\, Q(t, \tau) + R(t, 0, \tau) + \frac{\partial Q}{\partial t}(t, \tau) - \frac{\partial Q}{\partial \tau}(t, \tau) = P(t)\, D_1(t)\, Q(t, \tau)$$

$$2A(t)\, R(t, \tau, \rho) + \frac{\partial R}{\partial t}(t, \tau, \rho) - \frac{\partial R}{\partial t}(t, \tau, \rho) -$$

$$\frac{\partial R}{\partial \rho}(t, \tau, \rho) = Q(t, \tau)\, Q(t, \rho)\, D_1(t)$$

with boundary conditions (2.4). The solution of the problem posed is the function

$$P(t) = B_1^2(t)\, P_1(t), \quad Q(t, \tau) = -B_1(t)\, P_1(t)\, B^{\cdot}_1(t + \tau)$$

$$R(t, \tau, \rho) = P(t)\, B_1^{\cdot}(t + \tau)\, B^{\cdot}_1(t + \rho), \; 0 \leqslant t \leqslant T \; -h \leqslant \tau, \; \rho \leqslant 0$$

where $B_1(t)$, $P_1(t)$ are determined from the relations

$$B_1^{\cdot}(t) = -B_1(t + h)\, B(t + h), \; B_1(T) = 1, \; B_1(\sigma) \equiv 0, \sigma > T$$

$$P^{\cdot}_1(t) = -2A(t)\, P_1(t) + B_1^2(t)\, P_1^2(t), \; P_1(T) = H \qquad (4.3)$$

Equations (4.3) are easily integrable in explicit form similarly to (2.5), (2.6).

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